STATISTICAL ANALYSIS OF BASALTIC VOLCANISM NEAR THE YUCCA MOUNTAIN SITE

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GOALS

To estimate

1. the recurrence rate

2. the waiting time of the next eruption

3. the probability of at least one eruption during the next 10,000 years

4. the probability of volcanic disruption of the repository during the next 10,000 years (in progress)

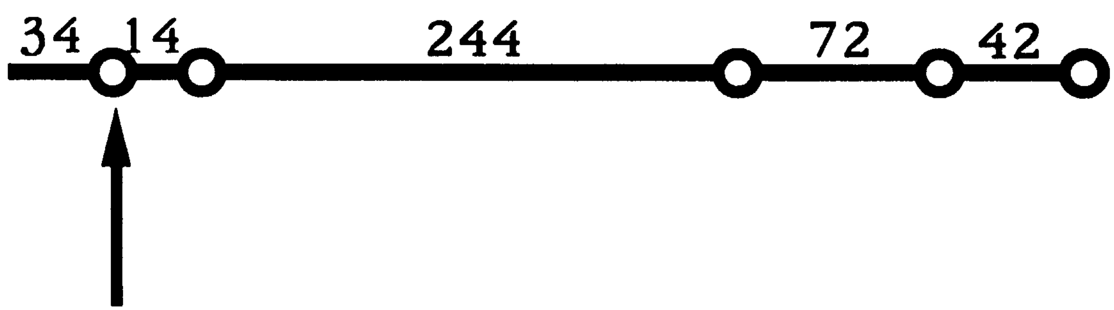
Need a model that captures the basic elements of the study :

1. **Objectivity**
2. **Trend**
3. **Predictability**
4. **Mathematical Simplicity**

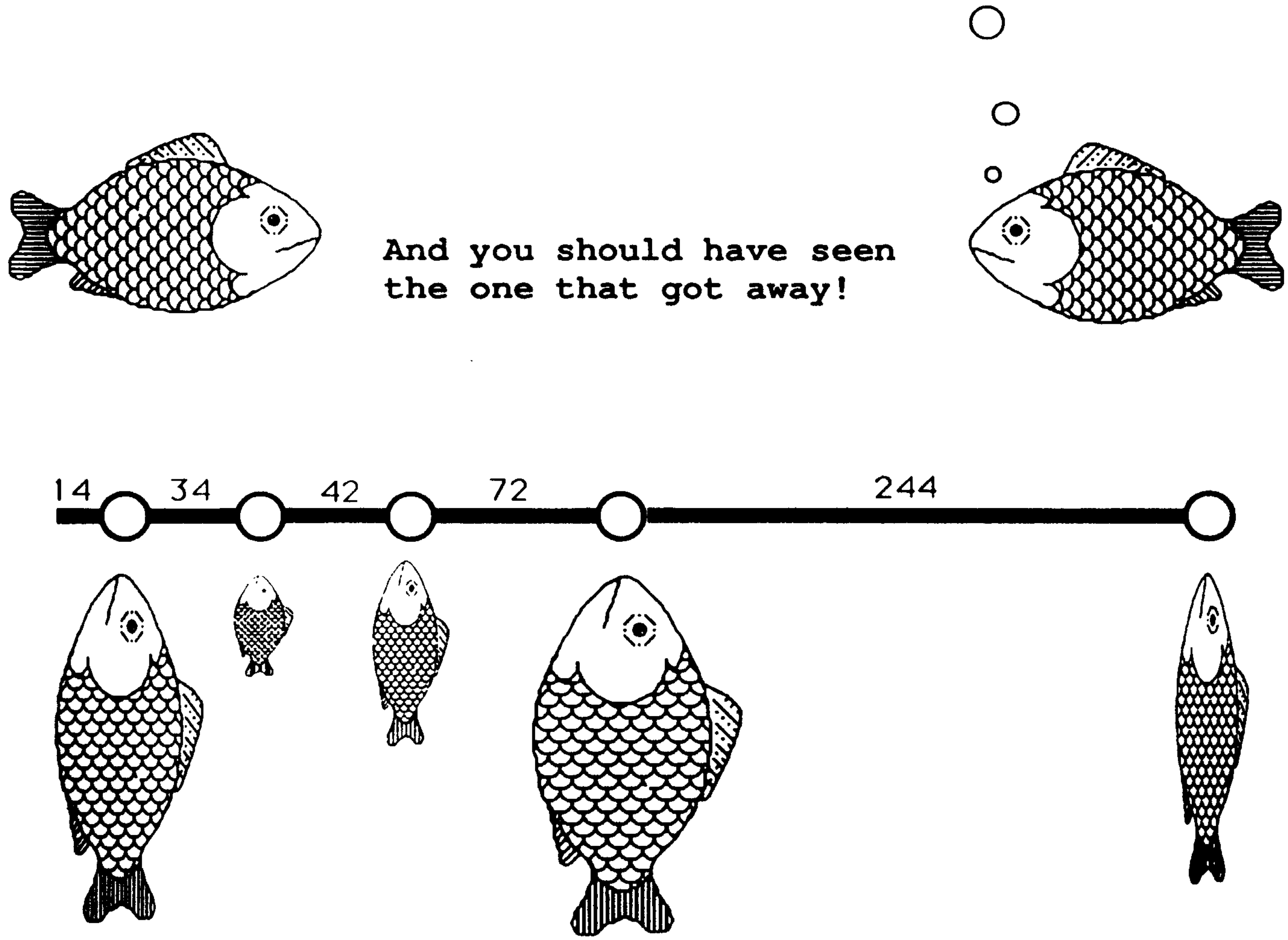
TIME SERIES

generated by stochastic phenomena (events)

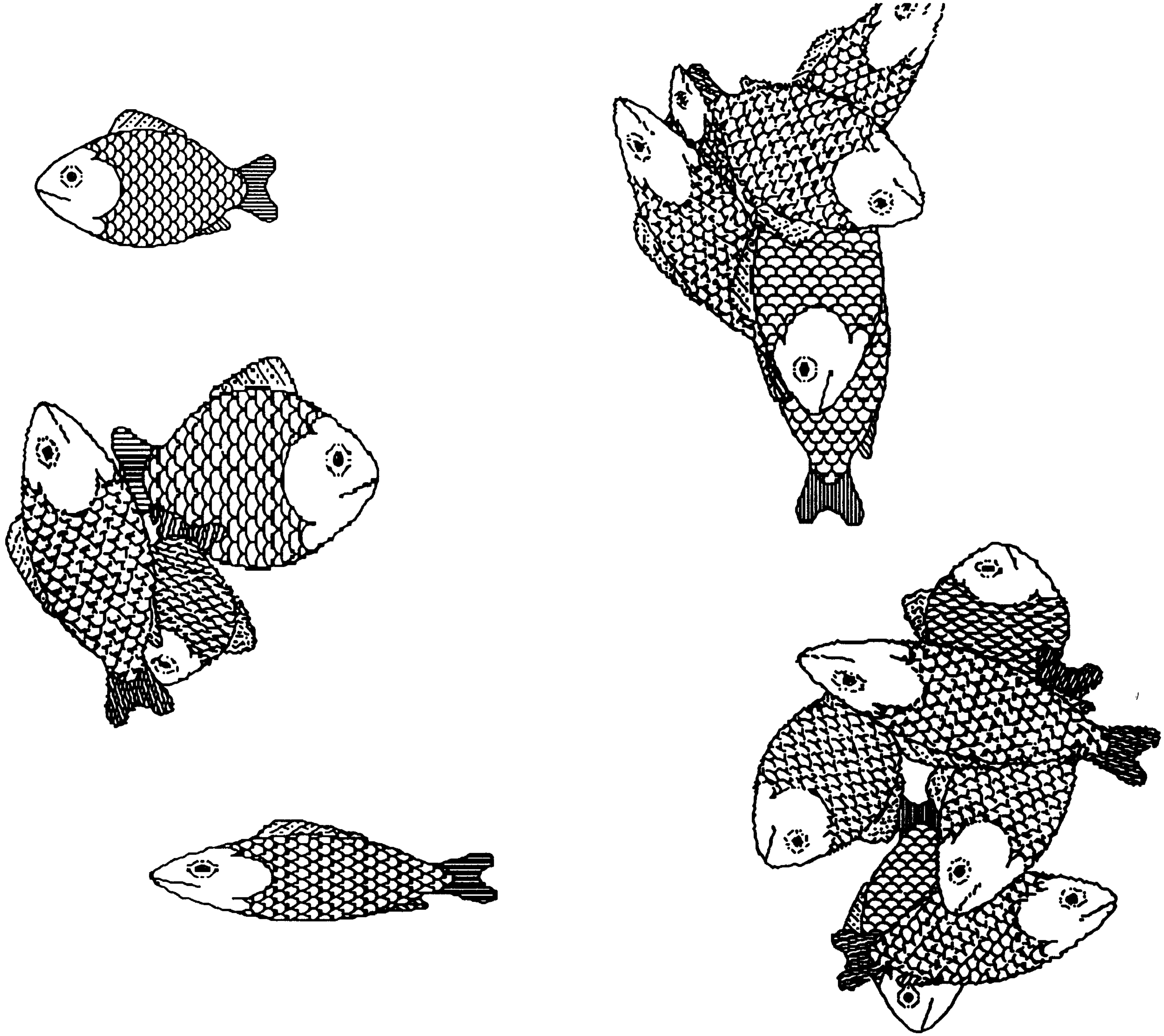
Data : 34, 14, 244, 72, 42 (inter-event times)



earthquake, volcanic eruption, etc.



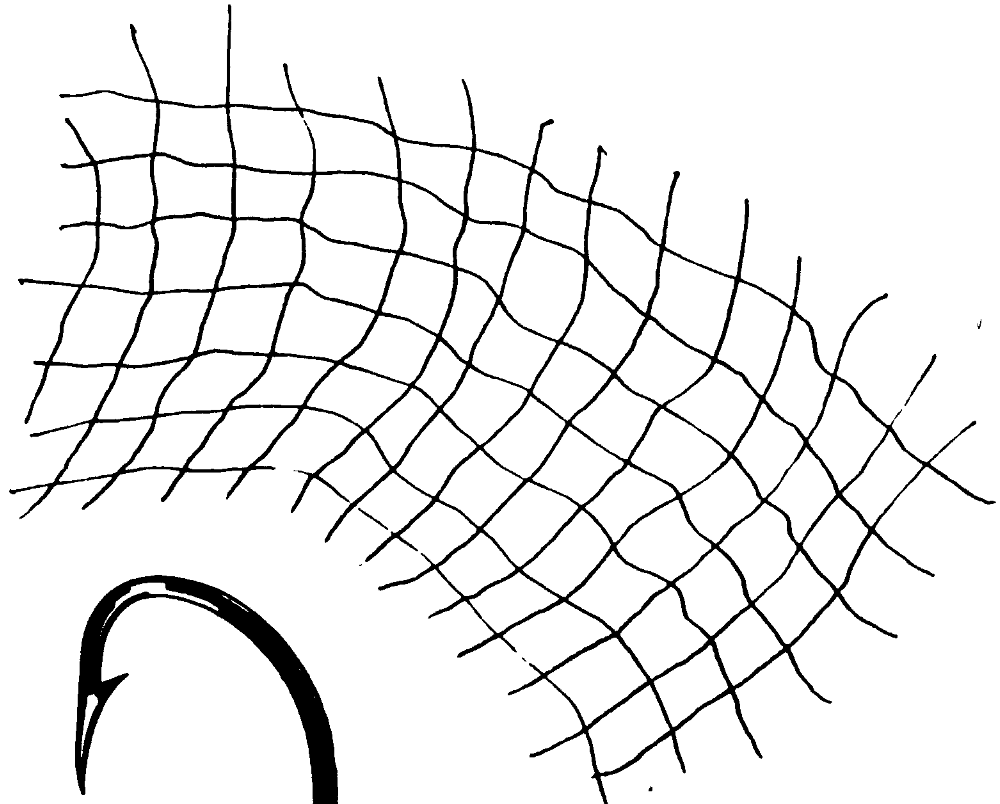
And you should have seen
the one that got away!
14
34
42
72
244



WHAT IS A SINGLE EVENT?

1. Need a clear definition

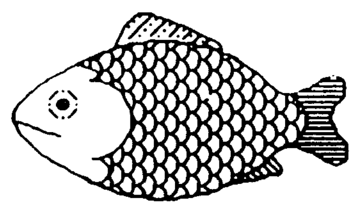
2. Based on the understanding of fishing techniques (or eruptive processes, etc.)



WHAT TO MEASURE?

- variables of interest

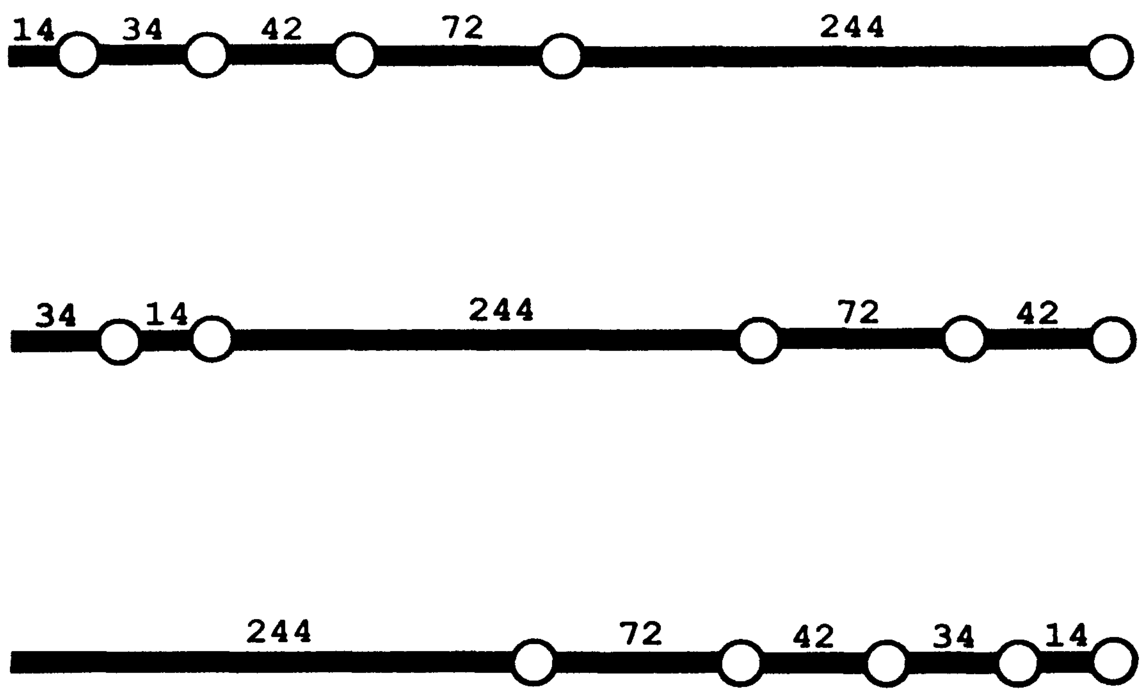
1. length
2. weight
3. volume
4. age
5. freshness

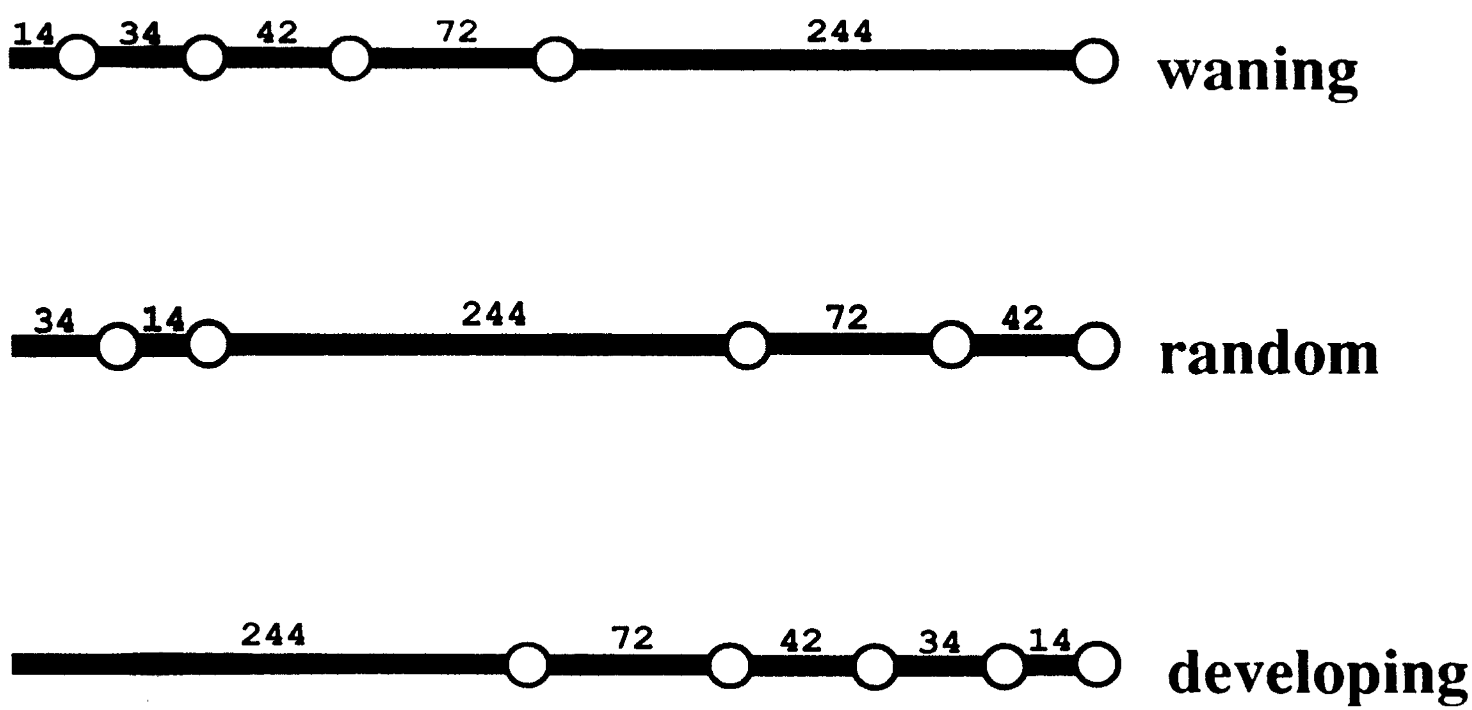


1. Define a single event
2. Measure each event
3. Count them all

→ generate a TIME SERIES

TIME TREND




14 34 42 72 244
waning
34 14 244 72 42
random
244 72 42 34 14
developing

A Simple Poisson Model

(a homogeneous Poisson Process, HPP)

ignores the time trend, and assumes

A CONSTANT RATE OF OCCURRENCES (λ).

λ = # events / obs. time

= reciprocal of average inter-event time

1. **GENERALIZE a constant λ with $\lambda(t)$, a function of time**

2. **Model $X(t)$ = number of events in $[0,t]$**

   $X(t)$ follows a nonhomogeneous Poisson process (NHPP) with parameter $\mu(t)$

$$\mu(t) = \int_0^t \lambda(s)\, ds$$

(Parzen, 1962, p. 138)

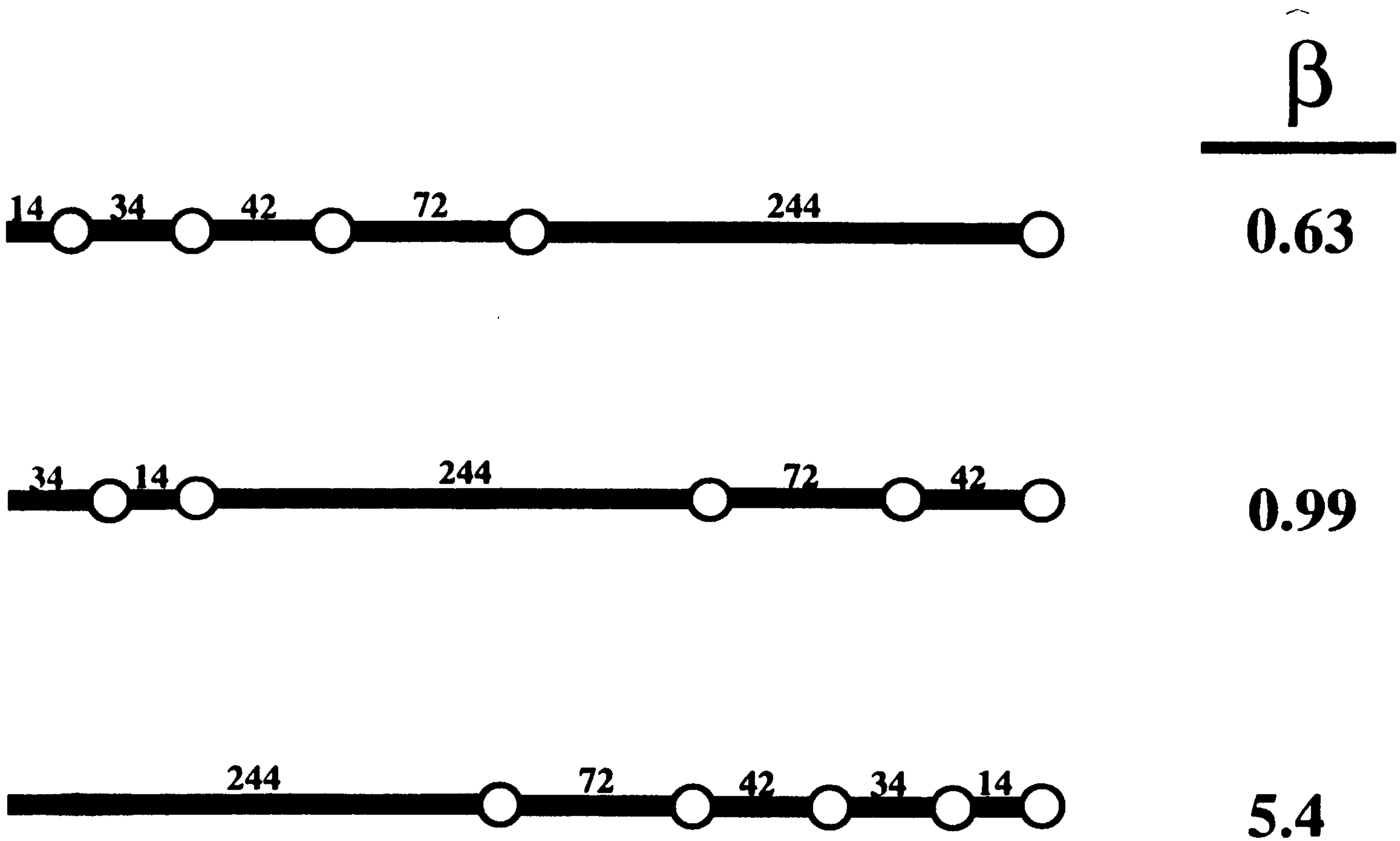
- **Choice of $\lambda(t) = (\beta/\theta)\,(t/\theta)^{\beta - 1}$**
- **yields $\mu(t) = (t/\theta)^{\beta}$**
- **implies a Weibull (θ,β)**

$$\beta \begin{cases} > 1 & \textbf{increasing} \\ = 0 & \textbf{simple Poisson} \\ < 1 & \textbf{decreasing} \end{cases}$$

Let $t_1, t_2, \ldots, t_n$ be the first n successive times of events in [0,t]: $t_1 < t_2 < \ldots < t_n$

- $\hat{\beta} = n / \sum_{i=1}^{n} \ln(t/t_i)$
- $\hat{\theta} = t/n^{1/\hat{\beta}}$
- $\lambda = (\hat{\beta}/\hat{\theta})\,(t/\hat{\theta})^{\hat{\beta}-1}$

(Crow 1974, 1982)


β̂
14 34 42 72 244
0.63
34 14 244 72 42
0.99
244 72 42 34 14
5.4

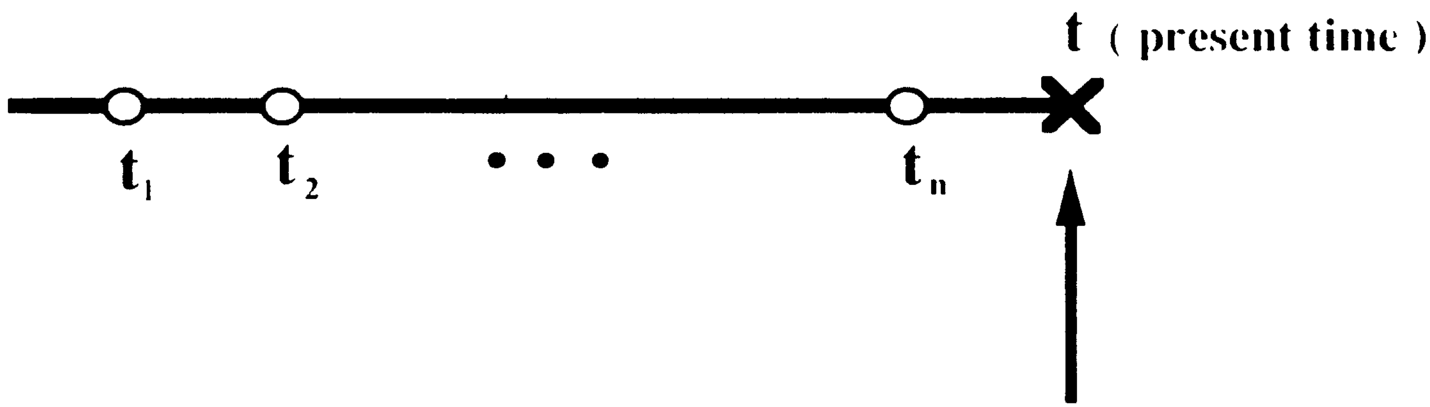
Goodness-of-fit test

$$H_0 : \beta = 1$$

$$H_A : \beta \neq 1$$
$$> 1$$
$$< 1$$

$$X^2 = 2n/\hat{\beta} \sim \chi^2(2n)$$

Instantaneous Recurrence Rate



$$\hat{\lambda}(t) = (\hat{\beta}/\hat{\Theta})(t/\hat{\Theta})^{\hat{\beta} - 1}$$

Volcanism Near the Yucca Mountain Site

t = ?

- Post-6 Ma (Pliocene and younger)
- Quaternary (< 1.6 Ma)

(Crowe et al. 1982, Smith et al. 1990, Wells et al. 1990)

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Identify a single event (eruption)

- cluster of centers (volcanic belt) ?
- a volcanic center ?
- a main cone ?
- a small vent ?

A main cone is the final stage of a single eruption, and a single eruption could have several small vents to accompany the main cone

(Crowe et al. 1983)

Count each widely recognized main cone as a single event, but do not require that the main cones in each center be of separate ages.

A. 3.7 Ma basalts

3.7 3.7 3.7 3.7

Daniel Feuerbach (personal communication 1990)

B. Buckboard Mesa

2.8

C. Red Cone,	1. 2
Northern Cone,	1. 2
Black Cone,	1. 2
Little Cones (2)	1. 2, 1. 2

(Vaniman et al. 1982)

D. Sleeping Butte Cones (2)

0. 28 0. 28

E. Lathrop Wells Cone

0. 01

(Crowe and Perry 1989, Wells et al. 1990)

Preliminary Data Set

3.7, 3.7, 3.7, 3.7, 2.8, 1. 2, 1. 2, 1. 2, 1. 2, 1. 2, 0. 28, 0. 28, 0. 01

(B) Quaternary

(A) Post-6 Ma

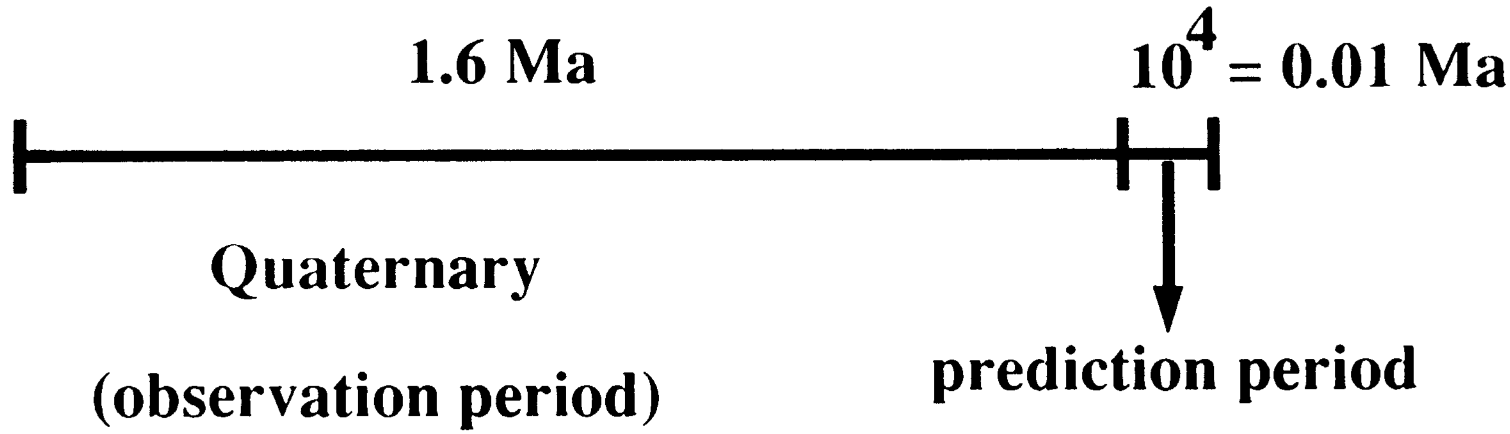
(A)
- $\widehat{\beta}$ = 2.29 (one-sided p-value $\doteq$ 0.005)
- $\widehat{\lambda}$ = 5×10^{-6} /yr

(B)
- $\widehat{\beta}$ = 1.09 (one-sided p-value $\doteq$ 0.45)
- $\widehat{\lambda}$ = 5.5×10^{-6} /yr

$\hat{\lambda} = 5.5 \times 10^{-6}/yr$

- **The estimated instantaneous recurrrence rate**

- **It represents the instantaneous eruptive status of the volcanism at the end of the observation time t (present)**



1. **The projected time frame is about 0.6% of the OP**
2. **It is only 5% of the average repose time**

$\Downarrow$

Suggests switching from a NHPP to a predictive HPP model

It is further justified on the basis of

1. **mathematical simplicity**
2. **objectivity (given the uncertainty of future geophysical phenomena)**
3. **a slight increasing trend ($\hat{\beta} = 1.09$ for the Quaternary volcanism**

Model $X(t_0)$ = # of eruptions during the next t_0 years.

$$X(t_0) \sim \text{Poisson}\,(\hat{\lambda} t_0)$$

Predictions

1. Average waiting time to the next eruption is $\hat{\lambda}^{-1}$

   (a confidence interval is possible)

2. Pr(at least one eruption during the next t_0 years)

   $= 1 - \exp\{\hat{\lambda} t_0\}$

Observation period	Empirical Results		Probability Isolation Period (yr)		
	$\widehat{\beta}$ (p-value)	$1/\widehat{\lambda}$ (Ma) (90% C.I.)	1	10^4	10^5
6.0 Ma-	2.29 (0.005)	0.20 (0.11, 0.45)	5×10^{-6}	0.05	0.39
1.6 Ma-	1.09 (0.45)	0.18 (0.08, 0.55)	5.5×10^{-6}	0.05	0.42

Polycyclic Volcanism

Lathrop Wells volcano is a polycyclic volcano

(Crowe et al. 1989, Wells et al. 1990)

One Step further: assuming there are 3 additional eruptions,

1.2, 1.2, 1.2, 1.2, 1.2, 0.28, 0.28, 0.01, 0.01, 0.01, 0.01

Then

1. $\hat{\beta} = 1.50$ (p-value = 0.125)
2. $\hat{\lambda} = 10^{-5}$/yr (doubled)
3. $\hat{\lambda}^{-1} = 9.7 \times 10^{4}$ years (50% sooner)