UNCERTAINTY: TYPES, QUANTIFICATION, AND DECISION

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The Coarsest Classification

Type 1: ***Uncertainty that reflects the variability in the outcome of a repeatable experiment***
("aleatory" uncertainty)

Examples:
- game of cards
- max annual wind speed

Attributes:
- objective (rel. freq. interpretation)
- independent of time
- can be quantified but not reduced by gathering information
- probability theory applies to it

Type 2: <u>Uncertainty from ignorance</u>
("epistemic" uncertainty)

<u>Examples:</u>

one of a kind

- does God exist?
- Is the accused innocent or guilty?
- when did the French revolution start?

- what is the conductivity of a given aquifer?
- is a fault seismically active?

- form and parameters of prob. distribution

subjective

<u>Attributes:</u>

- <u>it depends on available information</u>;

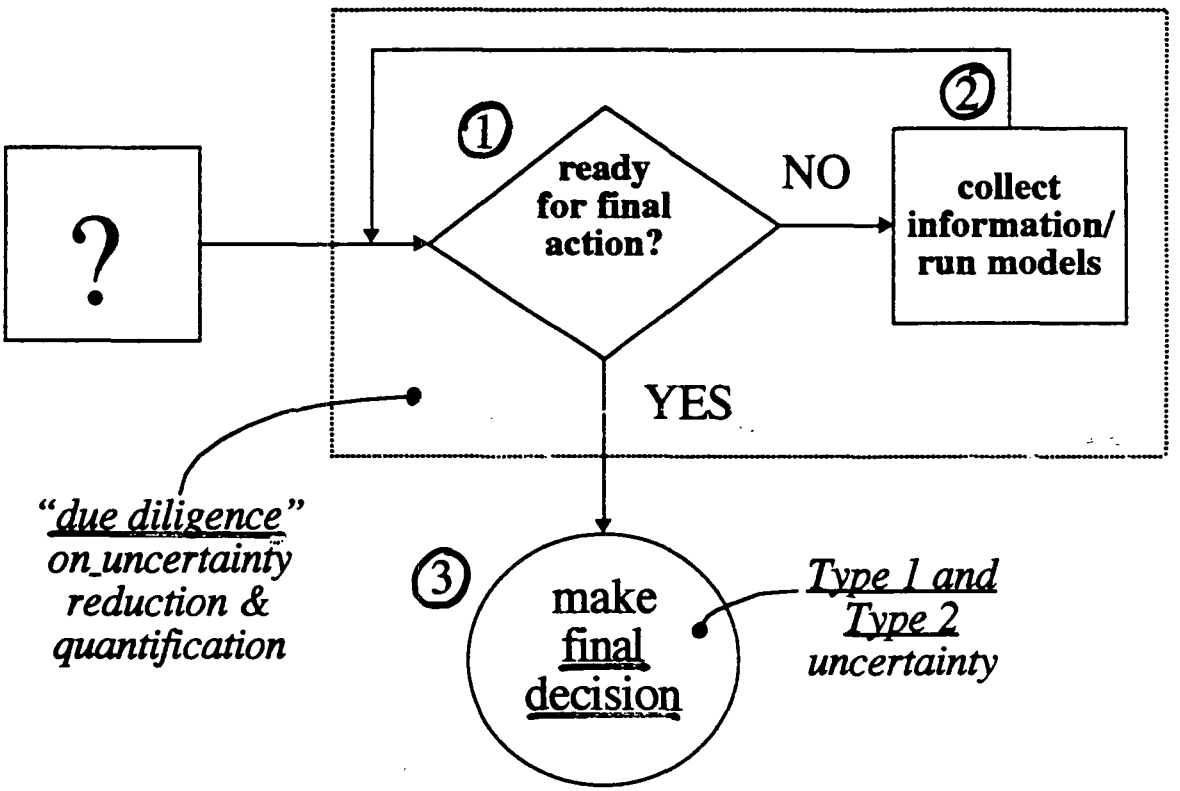
=> <u>it can be reduced by collecting data</u>
=> <u>usually varies in time</u>

- <u>it is often subjective</u> (degree of belief interp.)

=> probability theory may not apply
(in assessing uncertainty, individuals may not satisfy the axioms of PT)

- <u>sometimes it is objective</u> (stat. uncertainty)

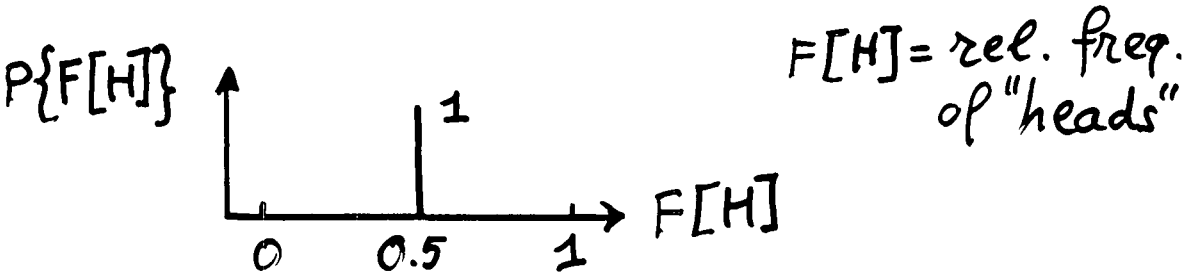
Does it Matter for Decision whether Uncertainty is of Type 1 or Type 2?



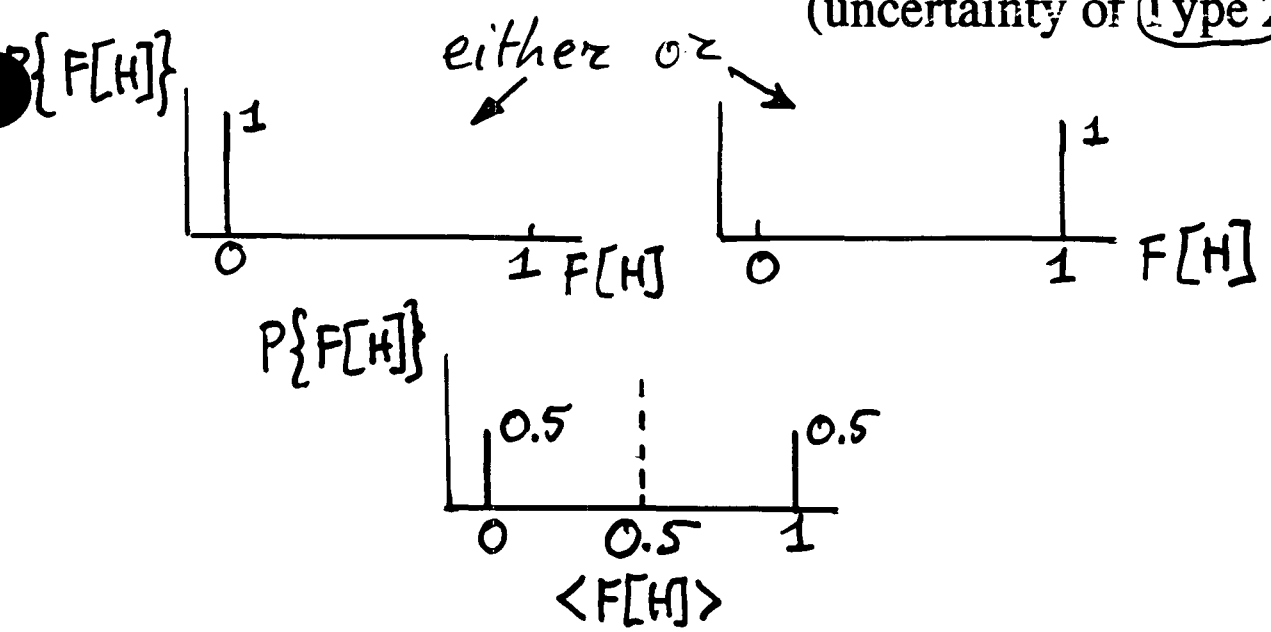
Final Decision

- ***Example.*** *Consider tossing a coin and compare two betting situations:*

1. bet before tossing, or for a randomly chosen toss, or for infinitely many tosses (uncertainty of Type 1)



2. bet after tossing, before the outcome is revealed (uncertainty of Type 2)



Rational people have the same betting attitude. Why?

- the mean value <F[H]> is the same.
- in (2), it is impossible to change the state of unc.

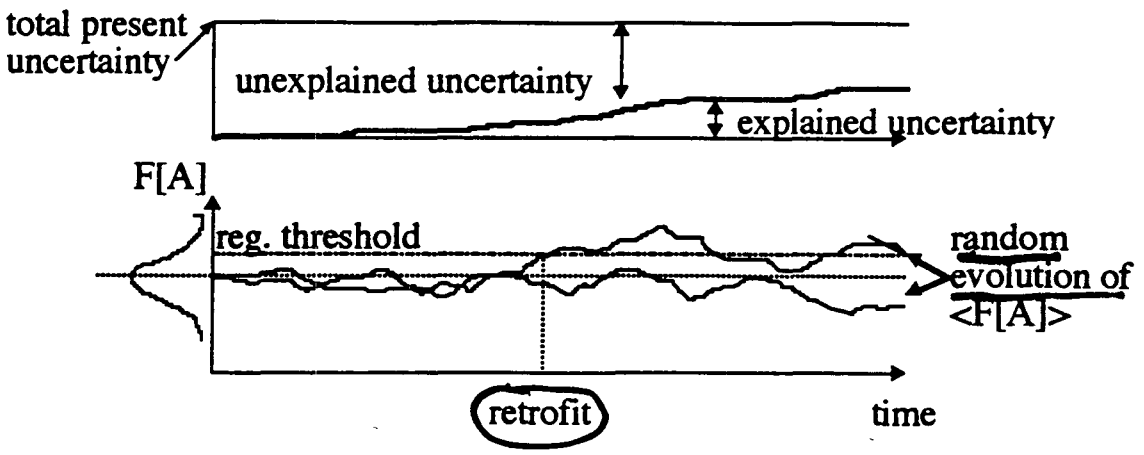
Final Decision (cont.)

More in general, suppose that:

- *the final decision depends on the relative frequency F[A] of an event A (e.g., A = release of hazardous substance).*
- *Due to ignorance, F[A] is uncertain.*

Decision theory says:

- If uncertainty on F[A] cannot change during the lifetime of the project, then all that matters is the mean value <F[A]>.
- If uncertainty on F[A] can change, then the temporal variability of <F[A]> should be considered.



Quantification of the Distribution of F[A] and the Mean Value <F[A]>

- *How can one quantify uncertainty on F[A]?*

- *Is there a simple way to obtain <F[A]>?*

Reasons for assessing uncertainty on F[A]

⇒ • **estimate <F[A]>** (and its possible temporal fluctuations)
• make sure no major source of uncertainty is neglected
• compare/communicate/document expert opinions
• identify areas where uncertainty can be reduced

Several methods, some formal, others informal

Uncertainty on F[A] and Value of <F[A]>

1. Standard Statistical Methods

- they apply when the information available is in the form of statistical samples (e.g. earthquake recurrence)
- for the most part, objective and established procedures

2. Methods Based on Expert Opinion/Model Runs

(a) Models as estimators (what if...). Model i produces $\hat{F}_i[A]$ (best-estimate models, bounding models, ...)

(a1) Formal Bayesian approach

- assign conditional distrib. of $\left\{\hat{F}_i[A]\right\}$ given F[A]
- find distribution of F[A] given $\left\{\hat{F}_i[A]\right\}$ using Bayesian analysis

(a2) Formal or informal judgmental approach

- judgmentally assess distrib. of F[A] given $\left\{\hat{F}_i[A]\right\}$

Example 1

- $\hat{F}_i[A] = F[A] + \varepsilon_i$

- ε_i iid normal, independent of F[A], with mean zero

$$\Rightarrow \boxed{< F[A] > \; = \; \frac{1}{n}\sum_{i=1}^{n} \hat{F}_i[A] \qquad \textbf{(mean rule)}}$$

Example 2

- $\ln\hat{F}_i[A] = \ln F[A] + \varepsilon_i$

- ε_i iid normal, independent of F[A], with mean zero

$$\Rightarrow \quad < \ln F[A] > \; = \; \frac{1}{n}\sum_{i=1}^{n} \ln \hat{F}_i[A]$$

For large n,

$$\boxed{< F[A] > \; = \; median\left\{\hat{F}_i[A]\right\} \quad \textbf{(median rule)}}$$

- **Depending on the relation between $\hat{F}_i[A]$ and F[A], other "rules" are possible.**

(b) ***models as (ME&CE) hypotheses*** (special case)

- assign probabilities $\{P_i\}$ to models, $\sum_i P_i = 1$

- then $\hat{F}_i[A]$ has probability P_i

$$\Rightarrow \boxed{< F[A] > \; = \; \sum_{i=1}^{n} P_i \cdot \hat{F}_i[A] \qquad \textbf{(mean rule)}}$$

Key Requirements for analysis (b):

- ***models must cover all possible interpretations***

- ***models must be exact (no approximation)***

In practice, these requirements are not enforced

MAIN POINTS

On Decision:

- What matters is the mean hazard $\langle F[A] \rangle$ and its future evolution

- The aleatory/epistemic distinction *per se* is not important

On the assessment of uncertainty on $F[A]$:

- The interpretation of models as alternative hypotheses is often incorrect and may produce erroneous values of $\langle F[A] \rangle$

- Bayesian methods are theoretically exact but often impractical

- Judgmental methods are less objective but simpler and often more accurate